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Analysis of Radar Return from Random
Surfaces Relative to Motion Between
Surface and Radar

by

R. L. Cosgriff

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## Department of ELECTRICAL ENGINEERING

# THE OHIO STATE UNIVERSITY
# RESEARCH FOUNDATION
## Columbus, Ohio

REPORT

by

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# ANALYSIS OF RADAR RETURN FROM RANDOM SURFACES RELATIVE TO MOTION BETWEEN SURFACE AND RADAR

## INTRODUCTION

In previous reports[1,2] the nature of signals received from radar was analyzed and the spectrum of the received signal, the IF signal, the video signal, and finally the intensity of the display were all determined but were limited to the case where no motion occurred between the reflecting surface and the radar.

In this report the same techniques are expanded so as to allow for the relative motion of radar and reflecting surface. Because the previous work associated with the receiver signals is still valid, major emphasis will be placed here upon the nature of the signal received from terrain.

In this development, the received signal from a point scatterer will first be determined for the special case of continuous (CW) radiation, then the pulsed case will be analyzed. The transform of this signal indicates the gain of a filter which, when excited by white noise, will produce a signal of the same statistical nature as that of the received signal for a collection of reflecting targets.

## THE CW CASE

The geometry to be considered is shown in Fig. 1. Here a single point radiator will be assumed to move past the antenna located at the origin. The location of this reflector will be given by

$$x = v_x t$$

$$y = y_o - v_y t.$$

The radius to the point scatterer is

$$r = (x^2 + y^2)^{\frac{1}{2}} = y_0 \left[ 1 - \frac{2v_y t}{y_0} + \frac{v_y^2 t^2}{y_0^2} + \frac{v_x^2 t^2}{y_0^2} \right]^{\frac{1}{2}},$$

which can be expanded to

$$r = y_0 - v_y t + v_x^2 t^2 / 2 y_0$$
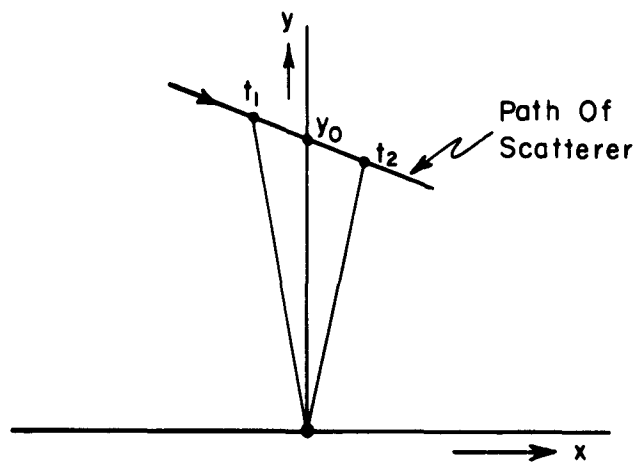
as a second-order approximation.



Fig. 1. Geometry of scatter as it moves through the antenna pattern.

Neglecting the $1/r^4$ range effect because of the small change of $1/r^4$ that occurs when the reflector is within the antenna beam-width $\Delta\theta$, the received signal will take the approximate form*

$$s_r = a_0 e^{j\omega_0 t} (e^{-j4\pi r/\lambda}) .$$

Substituting for $r$ yields

$$s_r = a_0 e^{j\left(\omega_0 + \frac{4\pi v_y}{\lambda}\right)t} e^{-j4\pi y_0/\lambda} e^{-j4\pi v_x^2 t^2 /2y_0\lambda}$$

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*Generally range effects are minimized by tapering the elevation pattern of the antenna involved.

which can be simplified to

$$s_r = a_o e^{-j\phi} e^{j(\omega' t - \kappa^2 t^2)}$$

where

$$\phi = 4\pi y_o / \lambda \qquad \omega' = \omega_o + 4\pi v_y / \lambda = \omega_o(1 + 2v_y/c)$$

$$\kappa^2 = \frac{2\pi}{\lambda} \frac{v_x^2}{y_o} \quad .$$

It is assumed that the antenna has a uniform pattern over azimuth angle $\Delta\theta$ so that the scatterer will be illuminated from $-t_1$ to $+t_1$, and that for time other than in this range $s_r = 0$.* Thus the Fourier transform of $s_r$ will become

$$S(j\omega) = \int_{-t_1}^{+t_1} e^{j[(\omega' - \omega)t - \kappa^2 t^2]} dt \quad .$$

Here $a_o$ is dropped since the final constant multiplier will be determined by the terrain in question. Note that $\phi$ has been neglected as $S(j\omega) S(-j\omega)$ will be of importance rather than $S(j\omega)$, and $e^{-j\phi}$ will drop out in this latter case. Rather than directly integrating, variables will be changed by first letting

$$t = t_o + \tau$$

where

$$t_o = (\omega' - \omega)/2\kappa^2 \quad ,$$

giving

$$S(j\omega) = a_o e^{+j(\omega' - \omega)^2/4\kappa^2} \int_{\tau = -t_1 - t_o}^{\tau = t_1 - t_o} e^{-j\kappa^2 \tau^2} d\tau \quad .$$

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*If the antenna is stationary, $t_1$ will be a function of range alone. But if the antenna is scanning, $t_1$ will be a function of azimuth and angular scan rate as we ᵗ.

Next $\kappa\tau$ is set equal to $Z\sqrt{\pi/2}$, with the result:

$$S(j\omega) = \frac{(a_o e^{+j(\omega'-\omega)^2 /4\kappa^2})\left(\sqrt{\frac{\pi}{2}}\right)}{\kappa} \int_{Z=\kappa\sqrt{\frac{2}{\pi}}(t_1 -t_o)}^{\kappa\sqrt{\frac{2}{\pi}}(+t_1 -t_o)} e^{-jZ^2 \pi/2}\, dZ.$$

This integral is now in standard form and can be evaluated by tables at the limits indicated. See Fig. 2 for evaluation of this integral.

If the transform of a single reflector is of the form indicated, the spectrum of the received signal $\Phi_r(\omega)$ from a collection of scatterers will be of the form

$$\Phi_{sr}(s) = \frac{B\frac{\pi}{2}}{\kappa^2}\left|\int_{\left(-t_1\kappa - \frac{\omega'-\omega}{2}\right)\sqrt{\frac{2}{\pi}}}^{\left(t_1\kappa - \frac{\omega'-\omega}{2\kappa}\right)\sqrt{\frac{2}{\pi}}} e^{-jZ^2 \frac{\pi}{2}}\, dZ\right|^2$$



CORNU'S SPIRAL

$$\int_o^w e^{-j\frac{\pi}{2}w^2}\, dw$$

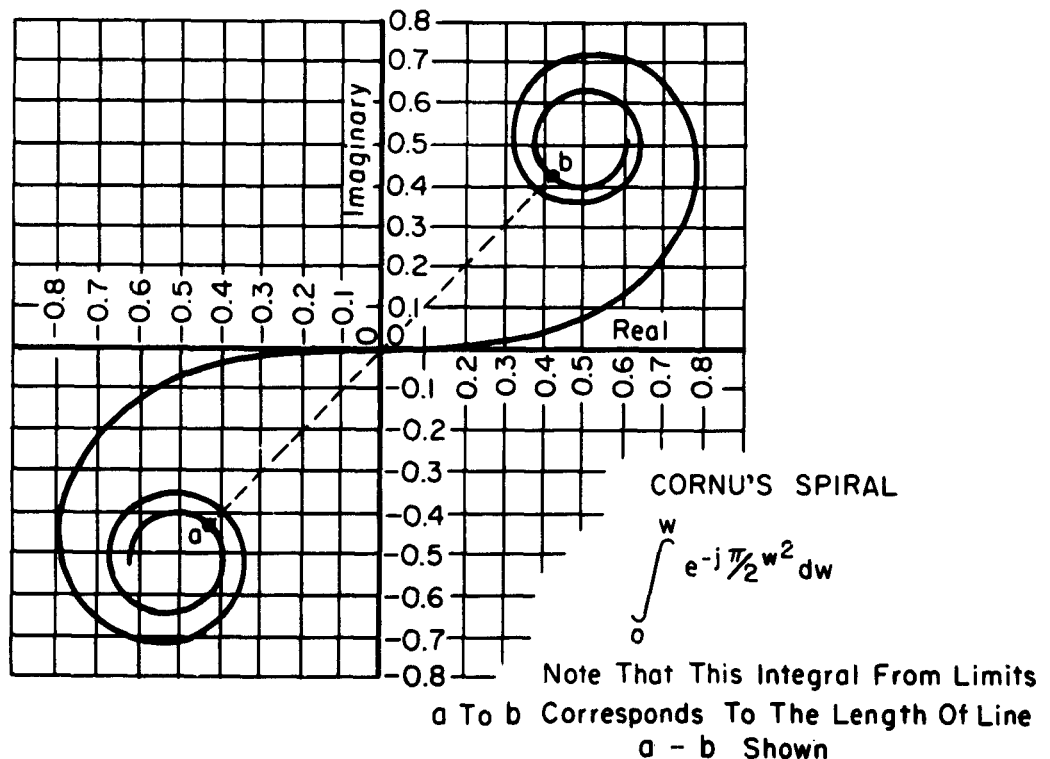Note That This Integral From Limits a To b Corresponds To The Length Of Line a - b Shown

Fig. 2.

where B is associated with the surface being illuminated. This spectrum will consist of a band centered at $\omega'$ and deviating from $\omega'$ by an approximate amount $\pm 2t_1 \kappa^2$ , namely, the instantaneous frequency deviation from $\omega'$ at the two time bounds.

## PULSED SYSTEM

In considering the pulsed case it is desirable, when possible, to relate the resulting spectrum to the spectrum for the CW case. It will be shown that the spectrum of the return signal is banded and each of these bands is similar to that of the single band of the CW system. For this reason constant reference will be made to the CW case to alleviate the computational difficulties involved.

In the pulsed situation it will be assumed that pulses of constant frequency are received centered in time at $t_n$, with

$$\omega' t_n + \kappa^2 t_n^2 = n K 2\pi$$

where K is an integer* and n is an integer corresponding to the $t_n$ that is associated with the midpoint of the $n^{th}$ pulse. Furthermore, the received pulse will be approximated by a constant-frequency signal equal to the instantaneous frequency at $t_n$, namely,

$$\omega_n = \omega' + 2 \kappa^2 t_n.$$

This pulse will be assumed to be on for a period given by the interval

$$t_n - \Delta t_n/2 \text{ to } t_n + \Delta t_n/2,$$

where

$$\Delta t_n \omega_n = k 2\pi$$

and k is the number of cycles in the pulse.

The transform of the $n^{th}$ pulse will be given by

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*K is the number of rf cycles between pulses.

$$e^{-j\omega t_n} \int_{-\Delta t_n/2}^{+\Delta t_n/2} e^{j(\omega_n - \omega)t} \, dt$$

$$= e^{-j\omega t_n} \left[ \frac{e^{j(\omega_r - \omega)\Delta t_n/2} - e^{-j(\omega_n - \omega)\Delta t_n/2}}{j(\omega_n - \omega)} \right] .$$

The bracketed term, equal to

$$\frac{2 \sin(\omega_n - \omega)\Delta t_n/2}{\omega_n - \omega} ,$$

is a broad envelope function which does not differ materially from pulse to pulse. It has a peak value of $\Delta t_n$ at $\omega = \omega_n$. Next the transform of the complete function will be determined for two cases, namely,

$$k = K, \text{ and } k \ll K.$$

For both cases the transform takes the simple form

$$S_{np}(j\omega) = 2 \sum \frac{e^{-j\omega t_n} \sin[(\omega_n - \omega)\Delta t_n/2]}{\omega_n - \omega} .$$

Here the exponential term merely inserts the proper delay time between pulses.

It is now necessary to inspect carefully $S_{np}(j\omega)$. If $k = K$, one band will exist which will be identical to that of the CW case, and for this reason the condition of $k = K$ will be considered as a guide. $S_{np}$ consists of a summation of terms, each delay term multiplied by

$$\frac{2 \sin(\omega_n - \omega)\Delta t/2}{\omega_n - \omega} .$$

If $K = k$, $S_{np}(j\omega)$ must equal $S(j\omega)$ as the pulse and CW signals are essentially equivalent. Thus

$$S_{np}(j\omega) = \left[ \frac{2 \sin\left((\omega' - \omega)\Delta t/2\right)}{\omega' - \omega} \right] \sum e^{-j\omega t_n}$$

$$= S(j\omega) \qquad \text{if } k = K$$
$$\Delta t = \frac{K 2\pi}{\omega'} .$$

Therefore, the form of the summation is given by

$$\sum e^{-j\omega t_n} = S(j\omega) \left[ \frac{(\omega' - \omega)}{2 \sin\left(\frac{(\omega' - \omega)}{\omega'} K\pi\right)} \right]$$

for $\omega$ in the vicinity of $\omega = \omega'$. Thus for $K \neq k$ the first band will be given by

$$S_{np}(j\omega) = S(j\omega) \frac{\sin\left(\frac{(\omega' - \omega)}{\omega'} k\pi\right)}{\sin\left(\frac{(\omega' - \omega)}{\omega'} K\pi\right)}$$

It is now necessary to determine the nature of $S_{np}(j\omega)$ for frequencies beyond the band of $S(j\omega)$. This will be done by considering the summation in detail. First it can be shown that $t_n$ is of the form

$$t_n = \frac{2\pi Kn}{\omega'} - \frac{4\pi^2 K^2 n^2 \kappa^2}{\omega'^3}$$

Likewise $\omega$ will be replaced by

$$\omega = \omega'\left(1 + \frac{\beta}{K}\right) + \Delta$$

where $\beta$ is a positive or negative integer. The reason for this latter substitution is that the center of the CW band occurs at $\omega'$ and the bands of the pulsed case will be centered at $(1 - \beta/K)\omega'$. Thus $\Delta$ is essentially the deviation of $\omega$ about the center of the $p^{th}$ band.

Consider a single term of the summation with $\beta = 0$, and also for $\beta$ arbitrary:

For $\beta = 0$,

$$e^{-j\omega t_n} = \exp\left\{-j\left[2\pi Kn - \frac{4\pi^2 K^2 n^2 \kappa^2}{\omega'^2} + \Delta\left(\frac{2\pi Kn}{\omega'} - \frac{4\pi^2 K^2 n^2 \kappa^2}{\omega'^3}\right)\right]\right\} ;$$

For $\beta \neq 0$,

$$e^{-jnt_n} = \exp\left\{-j\left[2\pi Kn\left(1 + \frac{\beta}{K}\right) - \frac{4\pi^2 K^2 n^2 \kappa^2}{\omega'^2}\left(1 + \frac{\beta}{K}\right)\right.\right.$$
$$\left.\left. + \Delta\left(\frac{2\pi Kn}{\omega'} - \frac{4\pi^2 K^2 n^2 \kappa^2}{\omega'^3}\right)\right]\right\} .$$

If these two terms can be related simply, all bands of the pulsed signal can be related to bands of the CW signal, as will be found to be the case. It is extremely important that each term of the two exponential functions just given be properly interpreted. First note that the first term, regardless of $\beta$, is a multiple of $2\pi$ and can thus be dropped for any $t_n$. The second term is a phase term, and the third term is the frequency-varying term expanded about $\omega'(1 + \beta/K)$. It should also be noted that each value of $\beta$ corresponds to one of the signal bands where $\beta$ is the number of bands from the zero band centered at $\omega'$.

If possible, it is desirable to drop the remaining $\beta/K$ factor of the second term, for in this case the two expressions will be the same indicating that all bands of $S_{np}(j\omega)$ are identical except for the envelope function. It can be demonstrated quickly that for a practical radar, $\beta/K$ the maximum value $<< 1$ ($\beta/K = 10^{-4}$, for example). However, a true measure of its significance for a single pulse will be the value of $4\pi^2 Kn^2 \kappa^2 \beta/\omega'^2$ . This term, if $0.1$ or less, can be neglected without further consideration. This restricted value of the term may or may not occur depending upon the configuration being considered. If this term cannot be neglected, $\kappa^2$ can be changed slightly to $\kappa'^2$ so that $\kappa^2 (1 + \beta/K) = \kappa'^2$ . It might then be argued that such a change might seriously modify the coefficient of $\Delta$ . However, this is not the case as the maximum deviation of this last term upon substitution of $\kappa'$ directly for $\kappa$ would be so small as to be of no significance. This change would be on the order of $10^{-6}$ radian or less.

It can be concluded that the first band $\beta = 0$ duplicates in shape the band of the CW case, and the remaining bands will be separated by $\beta/K\omega'$. For $\beta$ near zero, the bands are similar in shape to the zero band ($\beta = 0$). For large values of $\beta$ the band shapes may change from that of the zero-order band, but this shape can quickly be determined by substituting $\kappa'$ for $\kappa$ and using $\kappa'$ to replace $\kappa$ in the expression for the CW spectrum, which will then allow the determination of the band shape.

Thus the general expression for the summation in the vicinity of the $p^{th}$ band becomes

$$\sum e^{-j\omega t_n} = S_{\kappa'}\left(j(\omega'+\Delta)\right)\frac{\Delta}{2\sin\left(\frac{\Delta}{\omega'}\,K\pi\right)} \quad ,$$

where $\kappa$ involved in $S(j\omega)$ is replaced by $\kappa'$ and it is noted that $\kappa'$ is a function of $\beta$. Now note that this last summation is expressed in terms of $\Delta$, the deviation of $\omega$ about

$$\omega'\left(1 + \frac{\beta}{K}\right) \quad .$$

To align the two sides of the above terms frequency-wise $\Delta$ must be replaced by

$$\Delta = \omega - \omega'\left(1 + \frac{\beta}{K}\right) \quad ;$$

then for all bands,

$$\sum e^{-j\omega t_n} = \sum_{\beta=-\infty}^{+\infty} S_{\kappa'}\left(j\left(\omega-\omega'\frac{\beta}{K}\right)\right)\frac{\omega-\omega'(1+\beta/K)}{2\sin\left[\frac{(\omega-\omega'(1+\beta/K))K\pi}{\omega'}\right]}$$

where each $\beta$ corresponds to one of the frequency bands and is computable.

Finally $S_{np}(j\omega)$ can be expressed as

$$S_{np}(j\omega) = \left[\frac{\sin\left(\frac{(\omega'-\omega)}{\omega'}\,k\pi\right)}{\omega'-\omega}\right]$$

$$\left[\sum_{\beta=-\infty}^{+\infty} S_{\kappa'}\left(j\left(\omega-\omega'\,\frac{\beta}{K}\right)\right)\frac{\omega-\omega'(1+\beta/K)}{\sin\left[\frac{(\omega-\omega'(1+\beta/K))\,K\pi}{\omega'}\right]}\right] \quad .$$

Now it is necessary to properly interpret $S_{np}(j\omega)$. It consists of numerous separated bands, one band for each value of $\beta$. The peak value of each band will fall along an envelope at frequencies

$$\omega = \omega'(1 + \beta/K)$$

with an amplitude of

$$b\left(\frac{\sin \beta \, \frac{k}{K} \, \pi}{\beta \pi}\right)$$

where b is the maximum value of $S_{K'}(j\omega)$.

## CHARACTERISTICS OF RADAR

When relative motion occurs between terrain and a radar, filtering of various types has a pronounced effect upon the nature of the displayed image. First, if customary RF and IF stages are employed and these are followed by conventional detector and display, it is found that the display attenuates the terrain return fluctuations without seriously impairing the image as compared to that of a stationary radar display. The spectrum of the video signal is banded as shown in Figs. 3 and 4. Looking at one of the bands and the nature of the filtering accomplished by the display, the dissimilarity of the two cases is evident. First, note that if resolution is not to be impaired, the pass band associated with the display must be about the width of the spectrum associated with the stationary PPI radar and, as such, passes most of the fluctuations associated with the PPI display. On the other hand, the pass-bands pass only a small amount of the fluctuation associated with the motion scanning system; thus the fluctuations associated with this latter system are drastically attenuated.

To illustrate the limitations, consider the return video signal from two point scatterers in the case of stationary PPI and motional scanned systems when both are illuminated with five pulses (see Fig. 5). In (a) the intensity of return corresponding to each of the five traces is indicated for the motion-scanned case. In the case of the stationary PPI all traces would be identical, corresponding to any one of the traces shown in (a), and would depend upon orientation of the two points relative to the radar. For the motion-scanned case, each trace will differ and the average intensity, for the five traces would take the form shown in (b).

In concluding this section is it pointed out that the fluctuations will be attenuated in the display by roughly
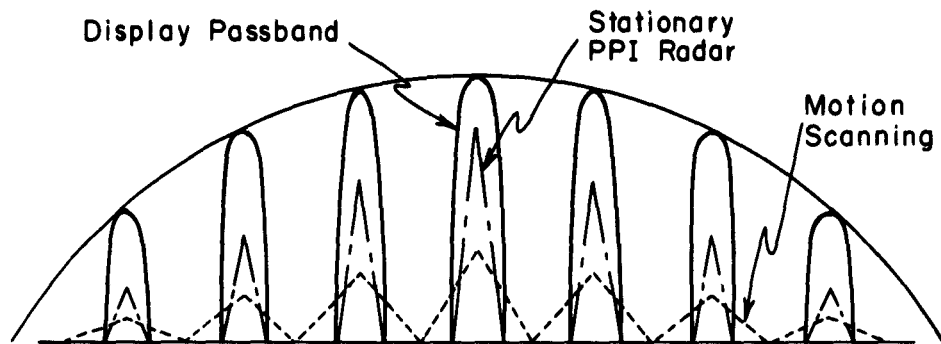
Fig. 3. Video spectra for stationary PPI scan and motion scanning
with pass bands associated with the display. Note that most
of the fluctuations involved in motion scanning fall outside
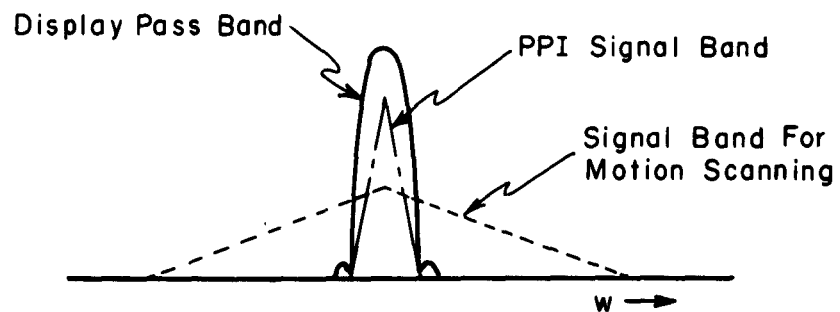the display pass band.



Fig. 4. Expanded scale of Fig. 3.

$$\left(2 - \frac{\omega_d}{2\omega_b}\right) \left(\frac{\omega_d}{2\omega_b}\right)$$

where $\omega_d$ is the width of the display pass bands and $\omega_b$ is the width of the signal bands before detection.

FILTERING IN THE IF

Filtering in the IF section, or filtering in the video if coherent video detectors are employed, drastically reduces the spectrum of the video signal. If filtering is such that the IF amplifier has a group of equally-spaced narrow IF bands as shown in Fig. 6, then frequencies outside these bands are rejected; moreover, signal components having these frequencies are due to reflectors whose doppler shifts are removed from the average doppler shift, that is, reflectors removed from the center of the antenna beam. Thus the return from terrain sectors not near the center portion of the antenna beam are effectively removed by such an IF filter. Therefore, the filter effectively narrows the beam width of the antenna. It should be noted that each of the multiple bands must be similarly filtered, that is, a comb-type filter must be employed.

Although it is possible to discuss in some detail the nature of the beam-narrowing effects in terms of the nature of the filter, this will not be undetaken at present. It is emphasized, however, that the antenna effectively looks as if it consists of separate transmitting and receiving antennas with the transmitting antenna having the pattern of the physical antenna and the receiving antenna generally appearing as a group of equally spaced antennas formed as an array, with each element the same as the physical antenna. In general the distribution of the field amplitudes of this collection of antennas is exponential in nature. Thus, in effect, a very special receiving antenna is under consideration. Side lobes will be determined primarily by the one-way pattern of the receiving antenna and not be the two-way pattern of the physical antenna.
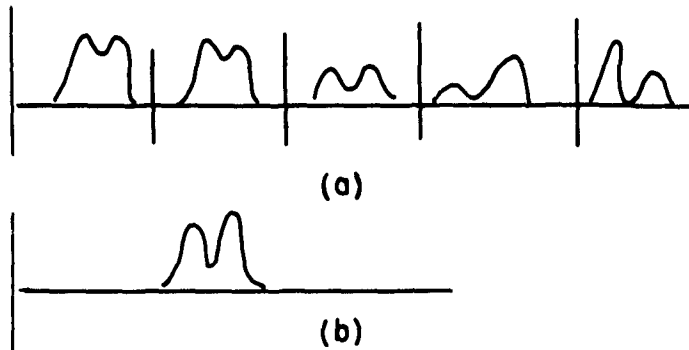
(a)

(b)

Fig. 5. (a) Four individual traces of motion
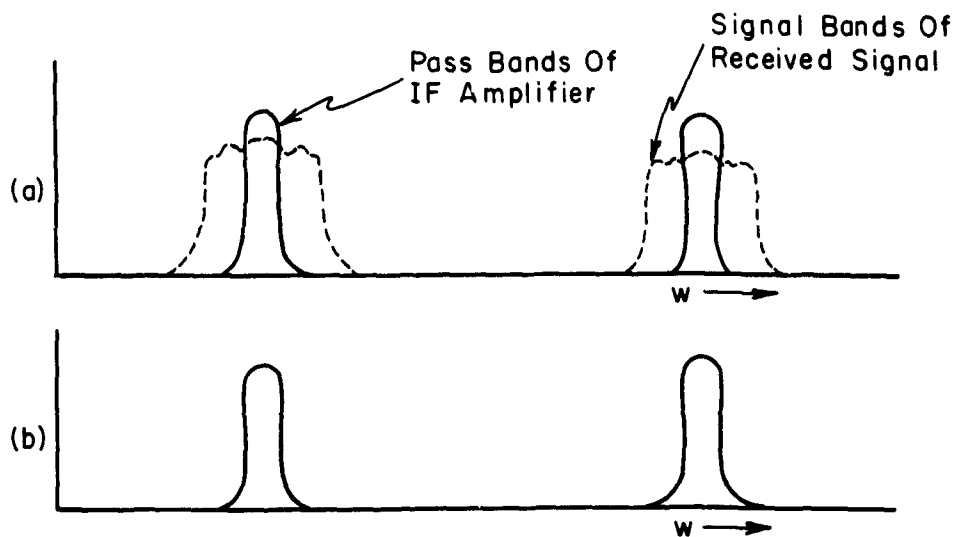scanned radar.
(b) Average of (a) as observed.



Fig. 6. (a) Spectrum of IF signal bands before filtering,
and pass bands of IF amplifier.
(b) Resultant IF signal bands.

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## CONCLUSIONS

The nature of the signal spectrum in various sections of a radar is formulated for the occurrence of relative motion between radar and terrain. The spectra of both the pulsed and CW systems have been determined with only minor approximations.

The effects of display filtering and of comb filtering in the IF have been shown, and the consequences outlined. The detailed nature of the displayed image and its limitations have not been included. These effects can be obtained using results given in the "Terrain Return Handbook."[3] for various types of displays.

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1. Cosgriff, R.L., "Analysis of the Radar Mapping Problem," Report 694-4, 17 September 1957, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 33(616)-3649, Air Research and Development Command, Wright Air Development Division, Wright-Patterson Air Force Base, Ohio.

2. Cosgriff, R.L., "A Study of Ground Mapping Radar," Report 694-5, 31 December 1957, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 33(616)-3649, Air Research and Development Command, Wright Air Development Division, Wright-Patterson Air Force Base, Ohio.

3. Cosgriff, R.L., Peake, W.H., and Taylor, R.C., "Electromagnetic Reflection Properties of Natural Surfaces with Application to Design of Radars and Other Sensors," (Terrain Handbook), Report 694-9, 1 February 1959, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 33(616)-3649, Air Research and Development Command, Wright Air Development Division, Wright-Patterson Air Force Base, Ohio.

4. Cosgriff, R.L., Peake, W.H., and Taylor, R.C., "Terrain Scattering Properties for Sensor System Design," (Terrain Handbook II), Engineering Experiment Station Bulletin 181, The Ohio State University, May 1960.